



IIT ROORKEE



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CERTIFICATION COURSE

EARTHQUAKE RESISTANT DESIGN OF FOUNDATIONS

Lecture 19: Dynamic Analysis of Piles under Vertical and Horizontal Vibrations

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ERD of Foundations - Modules

1. Introduction
2. Shallow Foundations
- 3. Pile Foundations**
4. Well Foundations
5. SSI for Deep Foundations



Module - 3

Pile Foundations

Module -3: Chapters

1. Piles for Static Vertical Load
2. Lateral Loads on Piles
3. Dynamic Analysis of Piles
4. Design for Lateral and Dynamic Loads
5. Codal Provisions

Chapter 1: Dynamic Analysis of Pile Foundation

- **Approaches**
- **Piles under Vertical Vibration**
- **Piles under Lateral Load**
 - Soil-Pile System
 - Natural Frequency of System
 - Method of Aseismic Analysis and Design of Piles
 - Numerical Example

Dynamic Analysis: Different Approaches

1. The soil is modeled as continuum

- I. Correctly represent geometrical damping as well as inertia of soil
- II. Soil medium is elastic, therefore nonlinearity of soil can not be considered

2. Soil Pile System is represented by a set of discrete (lumped) masses, spring and dashpot

- I. Geometrical damping is difficult to account
- II. Nonlinearity can be considered

3. Finite Element Analysis

This overcome the limitation of above approaches However at the expenses of high computation cost.

Piles Under Vertical Vibrations

Barkan (1962) proposed determination of soil pile stiffness from a cyclic vertical pile load test similar to a cyclic plate load test (Prakash and Puri, 1988). A plot of load P and elastic settlement z_1 may be represented by a straight line up to the working load in many situations. The constant of proportionality (k), the coefficient of elastic resistance of the pile is then:

$$P = kz_1 \quad (1)$$

where z_1 is elastic deflection of pile.

It represents the load required to induce a unit elastic settlement of the pile. The coefficient of elastic resistance of the pile depends on soil properties, pile characteristics (e.g., length), and the length of time the pile has been in the soil. For example, the elastic resistance of a pile may have different values during driving and some time later, particularly in soft clays.

Piles Under Vertical Vibrations (Continued...)

The natural frequency of the pile in vertical vibration is then given by;

$$f_{nz} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (2)$$

Based on the above simple concepts, Barkan (1962) described test data and typical values of elastic constants of piles and pile groups under both vertical and horizontal vibrations. This analysis does not consider damping in the system and the dynamics of the problem.

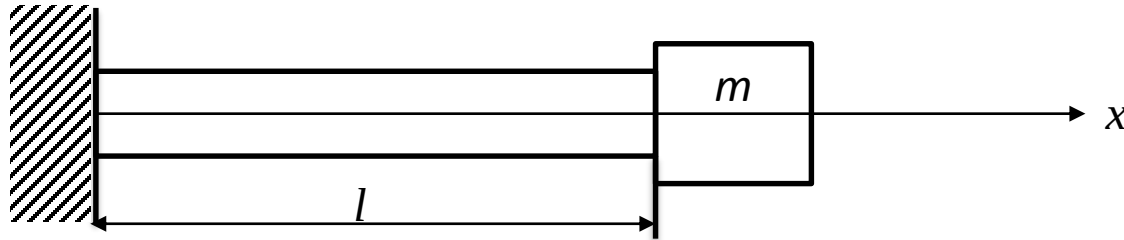
The soil modulus depends upon the strain in the soil. Therefore, k in equation (2) will have different values for machine foundation problems and for earthquake loading. No simple and direct relationship between strains in the soil along a pile, particularly in horizontal vibrations and soil deformations around the pile are available.

Since the elastic soil constants E , G , and k are strain or displacement dependent, the values of the elastic constant k determined from a lateral deflection of the order of 3.4mm in Barkan's test are not applicable to machine foundation problems.

Piles Under Vertical Vibrations (Continued...)

End-Bearing Piles

If piles are driven in soft soil and are embedded in sound rock or a hard stratum at their tip, the piles may be considered as end bearing piles. Deformations of the pile tip will not occur when dynamic loads are transferred to the pile. The pile may then be considered as an elastic rod fixed at its tip (base) and free at the top, with a mass m resting on the top (Figure).



Piles Under Vertical Vibrations (Continued...)

If no mass rests on top, we then have a solid resonant column with the fixed free condition, which has a resonant frequency given by (Prakash and Puri, 1988).

$$\omega_n = \frac{(2n-1)\pi v_r}{2l} \quad (3)$$

Where, ω_n = natural circular frequency

v_r = velocity of longitudinal wave propagation in the rod

l = length of the rod

for $n = 1$

$$\omega_n = \frac{\pi v_r}{2l} \quad \text{or} \quad f_n = \frac{v_r}{4l} = \frac{1}{4l} \sqrt{\frac{E}{\rho}} \quad (4)$$

where, f_n = natural frequency of rod in cycles per sec

E = Young's modulus of elasticity of the pile

$\rho = \frac{\gamma}{g}$ = mass density of the pile material

l = length of the pile material

Piles Under Vertical Vibrations (Continued...)

Displacement $u(= f (x , t))$ of a vibrating rod is given by

$$u = U(A \cos \omega_n t + B \sin \omega_n t) \quad (5)$$

For the case in which the weight of the pile is negligible as compared to the supported mass, the natural frequency may be obtained by applying the end condition

$$U = 0 \text{ at } x = 0$$

to a vibrating rod with zero weight. (Prakash and Puri, 1988). The displacement function of such a vibrating rod is given by:

$$U = D \sin \frac{\omega_n x}{v_r} \quad (6)$$

Where,

U = a function of x , which defines displacement

ω_n = natural circular frequency of rod

A, B, D = constants that depend on initial conditions

t = time

Piles Under Vertical Vibrations (Continued...)

Differentiating equation (5) with respect to x and t gives:

$$\frac{\partial u}{\partial x} = \frac{\partial U}{\partial x} = (A \cos \omega_n t + B \sin \omega_n t) \quad (7a)$$

and

$$\frac{\partial^2 u}{\partial t^2} = -\omega_n^2 U (A \cos \omega_n t + B \sin \omega_n t) \quad (7b)$$

For longitudinal excitation of the rod, displacement is zero at the fixed end. At the free end, a force that is equal to the inertia force of the concentrated mass is exerted on the rod. The equation of dynamic equilibrium may be written as

$$F = \frac{\partial u}{\partial x} AE = -m \frac{\partial^2 u}{\partial t^2} \quad (8)$$

Substituting equation (7) into equation (8), we get

$$AE \frac{\partial u}{\partial x} = m \omega_n^2 U \quad (9)$$

Finally, substituting U from equation (6) into equation (9), we get

$$AE \frac{\omega_n}{v_r} \cos \frac{\omega_n l}{v_r} = \omega_n^2 m \sin \frac{\omega_n l}{v_r} \quad (10a)$$

Piles Under Vertical Vibrations (Continued...)

Equation (10a) can be reduced to,

$$\frac{Al\gamma}{W} = \frac{\omega_n l}{v_r} \tan \frac{\omega_n l}{v_r} \quad (10b)$$

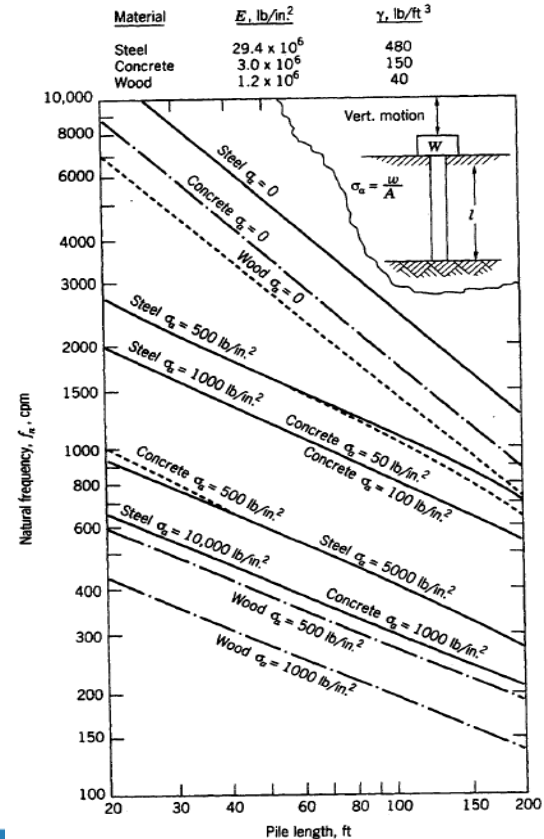
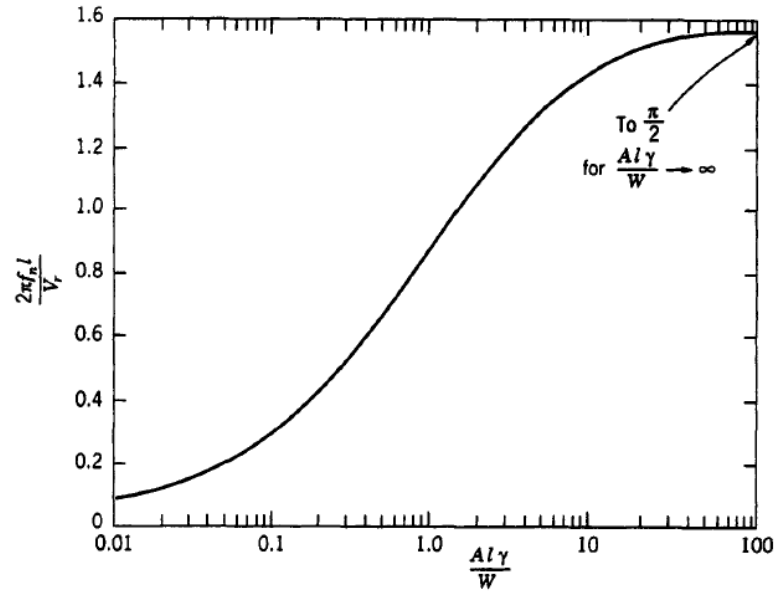
Where, $Al\gamma$ = weight of rod

W = weight of added mass

The solution of equation (10) is plotted in Figure, from which the natural frequency in vertical vibrations f_n may be determined.

The three curves in the upper part of the diagram illustrate the resonant frequencies of unloaded steel, concrete, and wooden piles, computed from equation (10). As the axial load is increased on a pile of given length, the resonant frequency is reduced

Piles Under Vertical Vibrations (Continued...)



Piles Under Vertical Vibrations (Continued...)

Friction Piles

In floating piles, unlike end-bearing piles, the load is transferred from the shaft to the soil, and their analysis under vertical vibrations is quite different than that for end-bearing piles. Some of the methods employed to determine the response of floating piles to vertical dynamic loads are as follows

1. A three-dimensional analysis (e.g., using the finite element method) considering the propagation of waves through the pile and soil
2. Solution of the one-dimensional wave equation, for example, in a manner similar to the solution of this equation to analyze the pile-driving process
3. An analysis of the response of a lumped mass-spring-dashpot system representing the pile and soil
4. An elastic analysis in which it is assumed that the elastic waves propagate only horizontally

Piles Under Vertical Vibrations (Continued...)

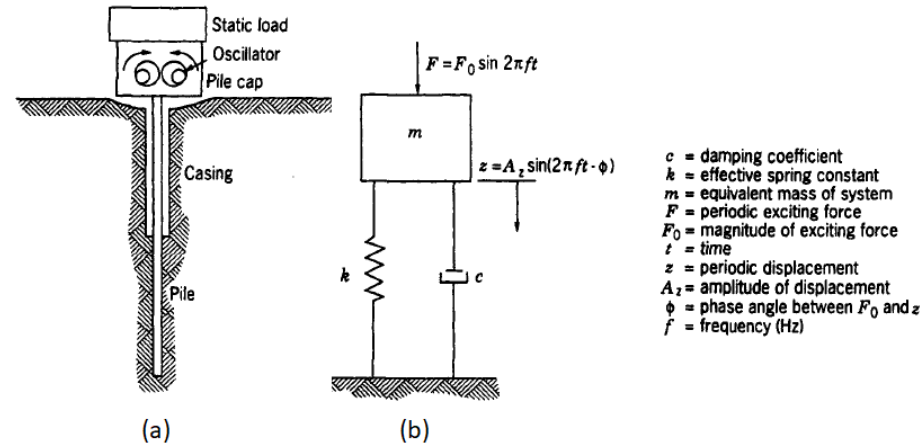
Maxwell's Lumped-Mass Spring-Dashpot Model

The vibrating pile is shown in Figure a and its single-degree-of-freedom model is shown in Figure b. With appropriate values of the mass, damping, and spring constant selected for the system, the foundation response can be determined from solutions of elementary theory of mechanical vibrations.

The solution for such a system is given by Prakash (1981) and Prakash and Puri (1988). In Figure, various terms have been defined.

$$Z_o = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \quad (11)$$

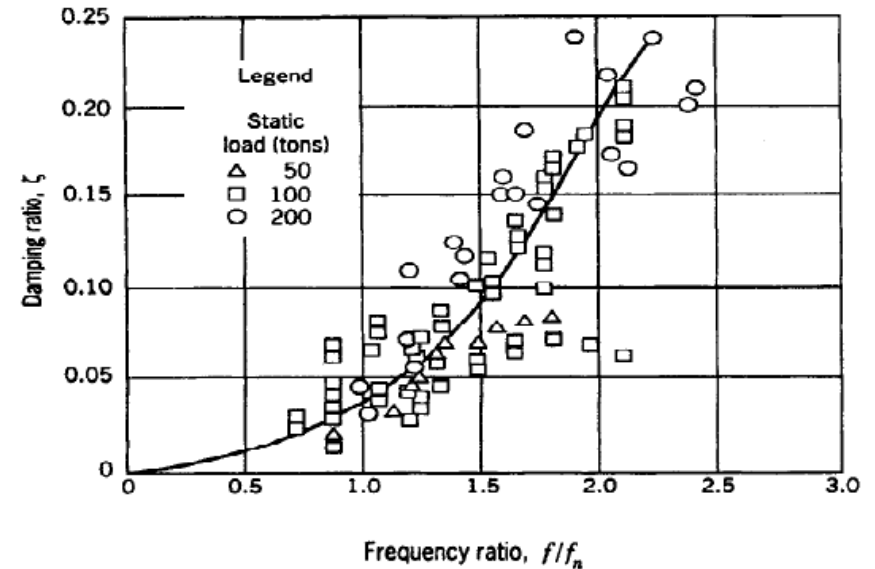
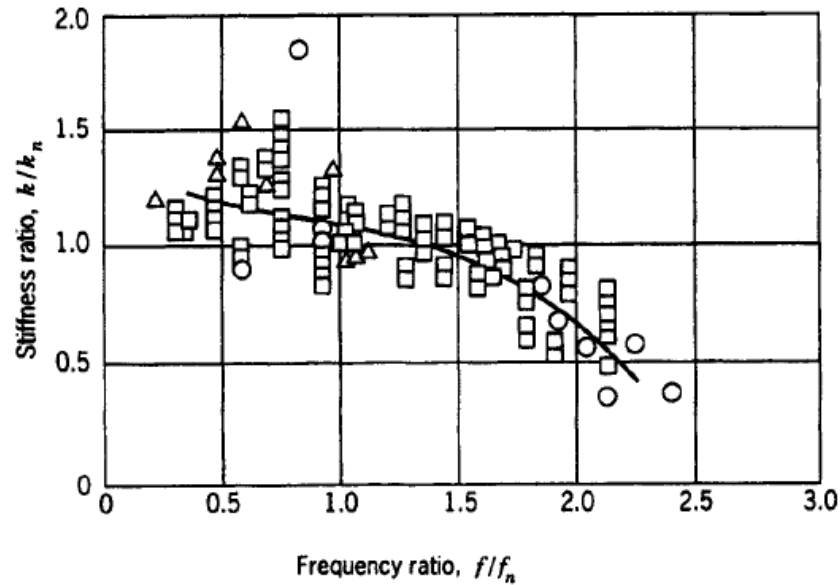
This solution differs from Barkan's solution since Maxwell et al. (1969) considered damping in the system.



Piles Under Vertical Vibrations (Continued...)

The computed damping ratio ξ for single piles was of the order of 0 to 0.04. A significant finding was that both the stiffness and the damping ratio varied with frequency. In particular, the response at resonance was not reliably predicted from data on stiffness and damping computed at non-resonant frequencies. The variation of stiffness, expressed in terms of a stiffness ratio k/k_n (where k_n = stiffness at resonant frequency), and damping ratio ξ with frequency ratio f/f_n , for pipe pile are plotted in Figure where f and f_n are operating and natural frequencies, respectively.

Piles Under Vertical Vibrations (Continued...)



Piles under Lateral Loads

- Soil-Pile System
- Natural Frequency of System
- Method of Aseismic Analysis and Design of Piles
- Numerical Example

Soil-Pile System

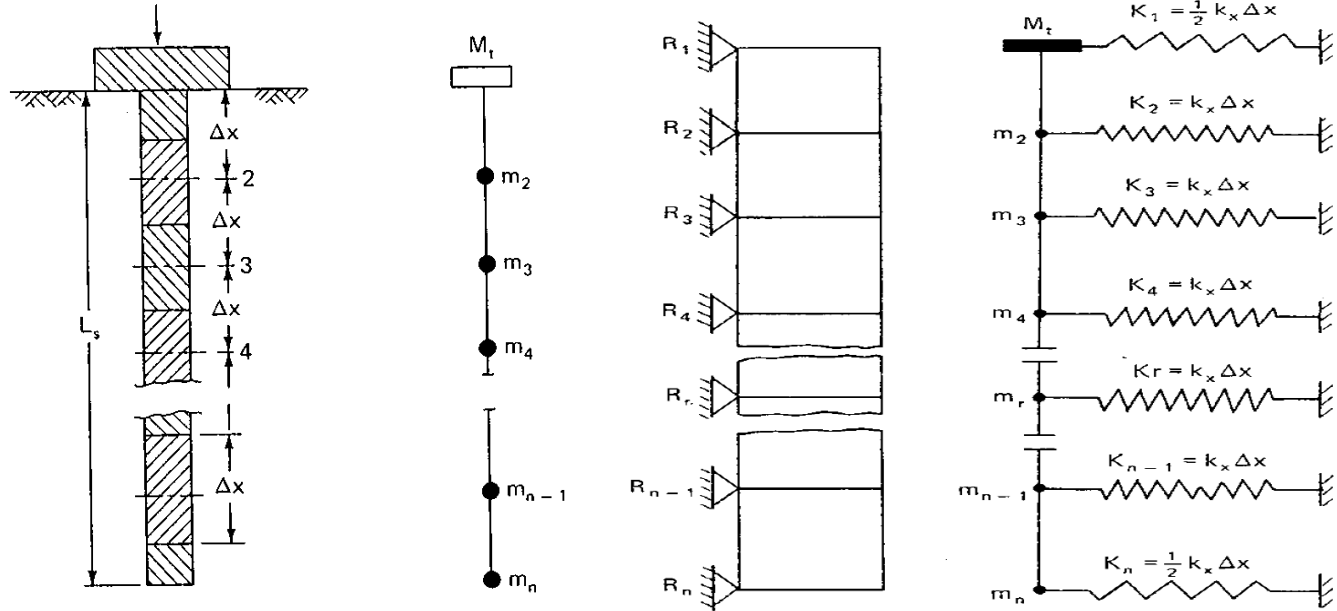
A reasonably practical solution for soil-pile interaction under dynamic loads has been proposed by Chandrasekaran (1974) (Prakash and Chandrasekaran, 1980). This analysis is based on the following assumptions:

1. The pile is divided into a convenient number of segments and mass of each segment is concentrated at its center.
2. The soil is considered as a linear Winkler's spring. The soil reaction is separated into discrete parts at the center of the masses. The soil modulus variation is considered both linearly varying with depth and constant with depth.
3. A fraction of the mass of the superstructure is concentrated at the pile top as M_t .
4. The system is one dimensional.
5. The pile end conditions are either completely free to undergo translation and rotation or completely restrained against rotation but free to undergo translation. Partial fixity at the top can be solved by interpolation.



Soil-Pile System (Continued...)

Assuming Soil Modulus Constant with depth



References

1. **Prakash S. (1981), "Soil Dynamics", McGraw-Hill Company, New York**
2. **Prakash S. and Sharma H.D., "Pile Foundations under Dynamic Loads",**

Thank You

